The Color of Whiteness and the Paradox of Diversity

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- Neil Altman (1) Email author (Altman.neil@gmail.com)
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Abstract

In this chapter, I seek to deconstruct the underlying structure of the notion of "diversity" as it is commonly used in the United States. Prejudice and ethnocentrism arise when Whiteness is the standard from which other ethnic and racial categories diverge and deviate. Mutually respectful interracial and intercultural communication and interaction depends on Whiteness taking its place as simply one among many racial and ethnic categories, all of which are socially constructed and none of which can be set up as the norm. Clinical implications are spelled out with an extended illustration.

Keywords

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Prerna Manik, Anshu Gupta and P. C. Jha

A Goal Programming Model for Advertisement Selection on Online News Media

Prerna Manik, Anshu Gupta and P. C. Jha

Abstract Promotion plays an important role in determining success of a product/service. Out of the many mediums available, promotion through means of advertisements is most effective and is most commonly used. Due to increasing popularity of the Internet, advertisers yearn for placing their ads on web. Consequently, web advertising has become one of the major sources of income for many websites. Several websites provide free services to the users and generate revenue by placing ads on its webpages. Advertisement for any product/service is placed on the site considering various aspects such as webpage selection, customer demography, product category, page, slot, time, etc. Further, different advertisers bid different costs to place their ads on a particular rectangular slot of a webpage, that is, many ads compete with each other for their placement on a specific position. Hence, in order to maximize the revenue generated through the ads, optimal placement of ads becomes imperative. In this paper, we formulate an advertisement planning problem for web news media maximizing their revenue. Mathematical programming approach is used to solve the problem. A case study is presented in the paper to show the application of the problem.

Keywords Advertisement planning · Revenue maximization · Online news web

P. Manik (✉) · P. C. Jha
Department of Operational Research, University of Delhi, Delhi, India
e-mail: prernamanik@gmail.com

P. C. Jha
e-mail: jhapc@yahoo.com

A. Gupta
SBPPSE, Dr. B. R. Ambedkar University, Delhi, India
e-mail: anshu@aud.ac.in

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1 Introduction

Advertising is an indispensable component of the marketing strategy for any firm. A well-designed advertisement campaign attracts a huge customer base, creates a brand name for the product and thereby enhances sales. Firms spend a large amount of capital to create effective exposure for its products by means of advertisements (ads). Today we see various media for advertising, starting from hoardings to television commercials, print ads to web media, and many more. Companies generally use a mix of various ad media to create maximum exposure for their products. Amongst all kinds of advertising media, the Internet has become the most famous and adopted media by the advertisers as well as consumers. And its popularity is increasing as information technology is reaching more and more people in the world and consumers stay connected to web for long hours. Other reasons for popularity of web advertising over other traditional media include traceability, cost effectiveness, reach, interactivity, etc. It is also capable of providing the dual features of both print and television media. The study in this paper focuses on a web ad scheduling problem where the objective is to maximize the revenue generated from placing ads on the multiple pages of a website. Maximization is achieved by selecting the ads on a slot from various competing ads in such a way that all the slots on every webpage under consideration are full at every time instant throughout the planning horizon. The proposed model surmounts one of the major limitations of the literature in the area.

Web ads commonly known as “banner ads” are devoted to promote, market, sell, or provide specific information about a product, service, or commercial event on web. Television and radio ads are expensive, short lived, and people tend to ignore them. However, in the case of web ads, customers have a choice as to whether or not they want to read or click on a web ad, while the ads may create an impression on the web users irrespective of their choice.

Though banner ads are the most popular ads on the web and constitute a major proportion of web advertising, there are also other ads that have been adopted by the advertisers such as are pop-up and pop-under ads, floating ads, unicast ads, etc. However, scope of this paper is limited to banner ads only. A banner ad is a small, typically rectangular, graphic image, which is linked to a target webpage. Many different types of banners with different sizes are being used in web advertisement. Rectangular-shaped banner ads are the most common type of banner ads. These banners usually appear on the side, top, or bottom of a screen as a distinct, clickable image [9]. For e.g., in Fig. 1, www.newswebsite.com displays a top banner ad of IBEF and two side banner ads of Artha Villas and CRAZEAL.

Due to constantly increasing popularity and power of web advertising, more and more websites and blogs are evolving that provide free services to their users. For e.g., websites such as Download.com, soft32, softpedia provide free software download facilities to the users. Also there are websites such as ApnaCircle and LinkedIn which help millions of professionals to connect and share their ideas for free. Then there are also websites such as hindustantimes.com and timesofindia.com, which provides their users a free service of e-paper, that is, a reader can read the newspaper

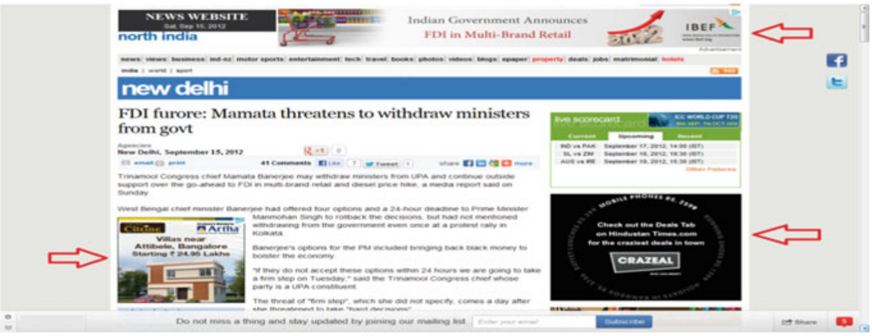

Fig. 1 Banner Ads on www.newswebsite.com (Date clicked: September 15, 2012)

online. Such websites generate major portion of their revenue by placing ads on their webpages. Hence, for such sites, optimal placement of ads on their webpages becomes imperative.

Many researchers have been working in the area of scheduling ads on web from past few years. One of the major focuses of the work has been on the effectiveness of web ads. Yager [14] described a general framework for the competitive selection of ads at web sites. A methodology was described in the paper for the use of intelligent agents to help in the determination of the appropriateness of displaying a given ad to a visitor at a site using very specific information about potential customers. Fuzzy system modeling was used for the construction of these intelligent agents. Dreze and Zufryden [3], Intern.com Corp. [5], Kohda and Endo [6], Marx [8] and Risden et al. [11] tackled the issue of increasing the effectiveness of web ads. Intern.com Corp. [5], McCandless [9], and Novak and Hoffman [10] described web advertising theories and terminologies. Researchers viz. Aggarwal et al. [2], Adler et al. [1], Kumar et al. [7] considered the issue of optimizing the ad space on the web. Aggarwal et al. [2] described a framework and provided an overview of general methods for optimizing the management of ads on web servers. They described a minimum cost flow model in order to optimize the assignment of ads to the predefined standard sizes of slots on webpages. Adler et al. [1] provided a heuristic called SUBSET-LSLF.

A major contribution in the area of ad scheduling has been done by Kumar et al. [7] and Gupta et al. [4]. Kumar et al. [7] addressed the problem of scheduling ads on a webpage in order to maximize revenue, for which they maximized the utilization of space available to place the ads. They used genetic algorithms to solve the problem. The major limitations of the model are that it considers only a particular side banner space on a specific page whose width is fixed and length can vary. The rectangular dimensions can thus be reduced to single dimension as the other dimension, i.e., width was assumed to be of unit size. All or some ads that can fit in this banner should therefore have the width of unit size. However, in practice banner ads that compete to be placed on a rectangular slot may be of varying rectangular dimensions. Second, in reality, the varying dimension of the slot for the banner can be a real value and need not necessarily be an integral multiple of the defined unit slot length. Therefore, the problem that maximizes the space utilization may not be the true representative of the revenue maximization problem. Gupta et al. [4] overcame the limitations of the model formulated by Kumar et al. [7]. They considered the set of ads competing to be placed on various rectangular slots (that may have varying rectangular dimensions) in a given planning horizon on various webpages of a news website in order to maximize the revenue, where the revenue is generated from the costs different advertisers pay to place their ads on the website. One of the limitations of the model formulated by Gupta et al. [4] is that it allows an ad to appear more than once on the same webpage. For instance, suppose that a webpage W_1 has three rectangular slots and suppose that an ad A_1 has appeared in slot 1 of this webpage at time period T_1. Then according to this model this ad A_1 can also appear in slot 2 and/or slot 3 of webpage W_1 at the same time period T_1.

In this paper, we formulate a web ad scheduling problem considering sets of ads competing to be placed on various rectangular slots (which may have different rectangular dimensions) in a given planning horizon on different webpages of a news website in order to maximize the revenue. The revenue is generated from the costs different advertisers pay to place their ads on the website. The proposed model restricts the selection of an ad on the same webpage more than once at any instant of time. We also discuss the solution methods for the proposed model, which is a 0-1 linear programming model. The model can be programmed and solved on LINGO [13] software. Depending on the available data the model may or may not be feasible. As the number of constraints increase the feasible area reduces and may tend to infeasibility. In this case, we use goal programming approach (GPA) [12] to obtain a compromised solution. The goal model of the problem can also be programmed and solved on LINGO [13].

The rest of the paper is organized as follows. In Sect. 2, we discuss the mathematical model formulation. A Case study has been discussed in Sect. 3. Section 4 concludes the paper.

2 Model Formulation

Notations

n : total number of webpages
m_j : number of rectangular slots on jth webpage
K : total number of ads
P : total number of time units in a day
Q : total number of days in a planning horizon
T : total number of time units over the planning horizon, where $T = P \times Q$
C_{ijk} : cost of kth ad competing for ith rectangular slot on jth webpage
S : set of K ads
S_{ij} : set of ads which compete for ith rectangular slot on jth webpage; $S_{ij} \subseteq S \forall i, j$

A_k : kth ad
w_k : minimum required time units for which kth ad appears in any rectangular slot
W_k : maximum time units for which kth ad appears in all the rectangular slots
D : total number of rectangular slots over a planning horizon (= Total number of rectangular slots on all the webpages × Length of planning horizon)

2.1 Web Ad Scheduling Problem

Web service providers endeavors to generate maximum revenue from the ads that are displayed on the webpages of their website. Therefore, optimal selection of the ads from the available sets of ads that compete to be placed on different rectangular slots of different webpages becomes critical.

We consider a set of K ads, $S = \{A_1, A_2, \ldots, A_K\}$ that compete to be placed on different rectangular slots of various webpages of a website in a planning horizon. The problem is formulated for a website consisting of n webpages, where jth webpage consists of m_j number of rectangular slots. A subset of ads S_{ij} competes to be placed on ith rectangular slot of jth webpage over a planning horizon. An advertiser k, where $A_k \in S_{ij}$, pays cost C_{ijk} to place his ad on ith rectangular slot of jth webpage with minimum frequency w_k and maximum frequency W_k.

Web ads are scheduled daily, fortnightly, weekly, monthly, or quarterly and so on depending on to the time units allocated to the ads. An ad which appears at any location stays there for some time and is then replaced by another ad. Consider for example, the minimum time for which an ad appears in any rectangular slot is one minute then, there will be $60 \times 24 = 1440$ time slots/units (P) in a day. And if the scheduling is to be done for say one week (i.e., $Q = 7$ days) then there will be a total of $1440 \times 7 = 10080$ time slots, i.e., the planning horizon would be $T = P \times Q = 1440 \times 7 = 10080$ time units.

Over a planning horizon, for each rectangular slot, web service provider selects ads which maximize their revenue and the unscheduled ads may compete for space in the next planning horizon with new ads. The set of ads assigned to all the slots for this time period is seen by the visitors who visit the site during that time interval and then the ads are updated according to their schedule. Now consider that we have in total $\sum_{j=1}^{n} m_j$ number of rectangular slots and a total of $T = P \times Q$ time units in the planning horizon, which can be considered as a scheduling problem of $D = T \times \sum_{j=1}^{n} m_j$ slots. Minimum frequency w_k represents the number of time units for which the ad A_k must appear when selected for some slot and maximum frequency W_k represents the number of time units for which the ad A_k must appear in all the rectangular slots over a planning horizon.

The problem to maximize the revenue generated by placing ads on the website over a planning horizon, which depends heavily on the costs different companies pay for placing their ads on ith rectangular slot of jth webpage is as follows:

$$\begin{aligned}
\textbf{Maximize}\quad & R=\sum_{i=1}^{m_j}\sum_{j=1}^{n}\sum_{k\in S_{ij}}\sum_{t=1}^{T}C_{ijk}x_{ijkt}\\
\textbf{Subject to}\quad & \sum_{t=1}^{T}x_{ijkt}\geq w_k z_{ijk}\quad \forall\, i,\, j, k\in S_{ij}\\
& \sum_{i=1}^{m_j}\sum_{j=1}^{n}\sum_{t=1}^{T}x_{ijkt}\leq W_k\quad \forall\, k\in S_{ij}\\
& \sum_{i=1}^{m_j}\sum_{j=1}^{n}\sum_{k\in S_{ij}}\sum_{t=1}^{T}x_{ijkt}\leq D\\
& \sum_{i=1}^{m_j}x_{ijkt}=1\quad \forall j, k\in S_{ij}, t\\
& \sum_{k\in S_{ij}}x_{ijkt}\leq 1\quad \forall\, i,\, j, t\\
& \sum_{k\in S_{ij}}z_{ijk}\geq 1\quad \forall\, i,\, j
\end{aligned} \tag{1}$$

where $x_{ijkt}=\begin{cases}1, & \text{if } k\text{th ad is chosen to be placed on } i\text{th rectangular slot of } j\text{th} \\ & \text{webpage at } t\text{th time unit}\\ 0, & \text{otherwise}\end{cases}$

$z_{ijk}=\begin{cases}1, & \text{if } k\text{th ad is placed on } i\text{th rectangular slot of } j\text{th webpage}\\ 0, & \text{otherwise}\end{cases}$

In the above problem $t = 1, \ldots, P \times Q = T$. Time slots are arranged in the ordinal manner i.e. the 1st P time units will correspond to 1st day, next P for 2nd day and so on.

Here, first constraint ensures that kth ad is assigned to at least w_k time slots. Second constraint guarantees that kth ad is assigned to not more than W_k number of slots over the planning horizon. Next constraint ensures the fullness of total number of rectangular slots over the planning horizon. Fourth constraint guarantees that if an ad is selected to be placed on any rectangular slot of a webpage at any given time period then that ad cannot appear on any other rectangular slot of that webpage at the same time unit. Next constraint ensures that at a particular time unit, on each rectangular slot on a webpage, not more than one ad can be placed. Last constraint ensures that number of times ad k appears on a particular rectangular slot over the planning horizon can be one or more than one.

Problem (1) can be solved using LINGO [13] software if a feasible solution to the problem exists. Otherwise for an infeasible solution, GPA [12] can be used to obtain a compromised solution.

2.2 Goal Programming Approach

In a simpler version of goal programming approach (GPA), management sets goals and relative importance (weights) for different objectives. Then an optimal solution is defined as one that minimizes both positive and negative deviations from set goals simultaneously or minimizes the amount by which each goal can be violated. First we solve the problem using rigid constraints only and then the goals of objectives are incorporated depending upon whether priorities or relative importance of different objectives are well defined or not. Problem (1) can be solved in two stages as follows:

$$\begin{aligned}
\textbf{Minimize} \quad & g_0(\eta, \rho, x, z) = \sum_{i=1}^{m_j} \sum_{j=1}^{n} \sum_{k \in S_{ij}} \eta_{ijk}^1 + \sum_{k \in S_{ij}} \rho_k^2 + \rho^3 \\
& \qquad + \sum_{j=1}^{n} \sum_{k \in S_{ij}} \sum_{t=1}^{T} \left(\eta_{jkt}^4 + \rho_{jkt}^4 \right) + \sum_{i=1}^{m_j} \sum_{j=1}^{n} \sum_{t=1}^{T} \rho_{ijt}^5 \\
& \qquad + \sum_{i=1}^{m_j} \sum_{j=1}^{n} \eta_{ij}^6 \\
\textbf{Subject to} \quad & \sum_{t=1}^{T} x_{ijkt} + \eta_{ijk}^1 - \rho_{ijk}^1 = w_k z_{ijk} \quad \forall\, i, j, k \in S_{ij} \\
& \sum_{i=1}^{m_j} \sum_{j=1}^{n} \sum_{t=1}^{T} x_{ijkt} + \eta_k^2 - \rho_k^2 = W_k \quad \forall\, k \in S_{ij} \\
& \sum_{i=1}^{m_j} \sum_{j=1}^{n} \sum_{k \in S_{ij}} \sum_{t=1}^{T} x_{ijkt} + \eta^3 - \rho^3 = D \\
& \sum_{i=1}^{m_j} x_{ijkt} + \eta_{jkt}^4 - \rho_{jkt}^4 = 1 \quad \forall\, j, k \in S_{ij}, t \\
& \sum_{k \in S_{ij}} x_{ijkt} + \eta_{ijt}^5 - \rho_{ijt}^5 = 1 \quad \forall\, i, j, t \\
& \sum_{k \in S_{ij}} z_{ijk} + \eta_{ij}^6 - \rho_{ij}^6 = 1 \quad \forall\, i, j \\
& \eta, \rho \geq 0
\end{aligned} \tag{2}$$

where, x_{ijkt} and z_{ijk} are as defined above and η and ρ are over-and under-achievement (positive- and negative-deviational) variables from the goals for the objective/constraint function and $g_0(\eta, \rho, x, z)$, is Goal objective function corresponding to rigid constraints.

The choice of deviational variable in the goal objective functions which has to be minimized depends upon the following rule. Let $f(X)$ and b be the function and its goal respectively and η and ρ be the over and under achievement variables then

if $f(X) \leq b$, ρ is minimized under the constraints $f(X) + \eta - \rho = b$,
if $f(X) \geq b$, η is minimized under the constraints $f(X) + \eta - \rho = b$,
if $f(X) = b$, $\eta + \rho$ is minimized under the constraints $f(X) + \eta - \rho = b$.

Let $(\eta^0, \rho^0, x^0, z^0)$ be the optimal solution for the problem (2) and $g_0(\eta^0, \rho^0, x^0, z^0)$ be its corresponding objective function value then finally GP problem can be formulated using optimal solution of the problem (2) through the problem (1) as follows:

$$
\begin{aligned}
\textbf{Minimize} \quad & g(\eta, \rho, x, z) = \eta^7 \\
\textbf{Subject to} \quad & \sum_{t=1}^{T} x_{ijkt} + \eta^1_{ijk} - \rho^1_{ijk} = w_k z_{ijk} \quad \forall\, i, j, k \in S_{ij} \\
& \sum_{i=1}^{m_j} \sum_{j=1}^{n} \sum_{t=1}^{T} x_{ijkt} + \eta^2_k - \rho^2_k = W_k \quad \forall\, k \in S_{ij} \\
& \sum_{i=1}^{m_j} \sum_{j=1}^{n} \sum_{k \in S_{ij}} \sum_{t=1}^{T} x_{ijkt} + \eta^3 - \rho^3 = D \\
& \sum_{i=1}^{m_j} x_{ijkt} + \eta^4_{jkt} - \rho^4_{jkt} = 1 \quad \forall\, j, k \in S_{ij}, t \\
& \sum_{k \in S_{ij}} x_{ijkt} + \eta^5_{ijt} - \rho^5_{ijt} = 1 \quad \forall\, i, j, t \\
& \sum_{k \in S_{ij}} z_{ijk} + \eta^6_{ij} - \rho^6_{ij} = 1 \quad \forall\, i, j \\
& \sum_{i=1}^{m_j} \sum_{j=1}^{n} \sum_{k \in S_{ij}} \sum_{t=1}^{T} C_{ijk} x_{ijkt} + \eta^7 - \rho^7 = R^* \\
& g_0(\eta, \rho, x, z) = g_0(\eta^0, \rho^0, x^0, z^0) \\
& \eta, \rho \geq 0
\end{aligned}
\tag{3}
$$

where R^* is the aspiration level desired by the management on revenue and $g(\eta, \rho, x, z)$ is objective function of the problem (3). Problem (3) is solved using LINGO [13].

3 Case Study

In case of online news services, users spend long time on sites for reading news. In case of such websites, ads are updated periodically during this period, which is taken to be 1 h (length of one time slot) here.

We consider a news website which consists of five webpages. These pages have 3, 4, 2, 3, and 3 rectangular slots, respectively. A set of sixty ads, $S = \{A_1, A_2, \ldots, A_{60}\}$ compete to be placed on webpages of a news website in a planning horizon, which is taken as 1 week. Now, a week consists of 7 days and each day consists of 24 h. Since ads are updated every hour on the webpages, we refer to each hour as a time unit. Thus, in this case, we have $168(= 24 \times 7)$ time units to schedule ads. Ads need to be placed in $D = \sum_{j=1}^{5} m_j \times T(= 3 + 4 + 2 + 3 + 3) \times 168 = 2520$ slots. Sets of ads competing for ith rectangular slot on jth webpage and corresponding costs are as follows:

$S_{11} = \{A_1, A_3, A_6, A_8, A_{10}, A_{13}, A_{15}, A_{18}, A_{20}, A_{23}, A_{25}, A_{27}, A_{29}, A_{32}, A_{33}, A_{36}, A_{38}, A_{40}, A_{43}, A_{44}, A_{46}, A_{49}, A_{51}, A_{54}, A_{57}, A_{59}\}; C_{11k} = 2,500 \quad \forall k \in S_{11}$

$S_{21} = \{A_2, A_4, A_6, A_9, A_{11}, A_{13}, A_{14}, A_{16}, A_{19}, A_{22}, A_{24}, A_{26}, A_{28}, A_{31}, A_{33}, A_{35}, A_{37}, A_{39}, A_{42}, A_{44}, A_{47}, A_{49}, A_{50}, A_{52}, A_{54}, A_{56}, A_{57}, A_{60}\}; C_{21k} = 2,300 \quad \forall k \in S_{21}$

$S_{31} = \{A_1, A_2, A_5, A_8, A_{10}, A_{12}, A_{14}, A_{17}, A_{18}, A_{20}, A_{23}, A_{24}, A_{26}, A_{28}, A_{29}, A_{31}, A_{34}, A_{36}, A_{37}, A_{38}, A_{40}, A_{42}, A_{43}, A_{45}, A_{47}, A_{50}, A_{51}, A_{53}, A_{55}, A_{58}, A_{59}\}; C_{31k} = 2,000 \quad \forall k \in S_{31}$

$S_{12} = \{A_2, A_3, A_5, A_7, A_9, A_{11}, A_{13}, A_{15}, A_{17}, A_{19}, A_{21}, A_{22}, A_{24}, A_{25}, A_{27}, A_{30}, A_{32}, A_{35}, A_{36}, A_{38}, A_{39}, A_{41}, A_{43}, A_{44}, A_{46}, A_{47}, A_{49}, A_{52}, A_{54}, A_{56}, A_{60}\}; C_{12k} = 1850 \quad \forall k \in S_{12}$

$S_{22} = \{A_1, A_3, A_4, A_6, A_8, A_{11}, A_{14}, A_{16}, A_{18}, A_{19}, A_{21}, A_{23}, A_{26}, A_{27}, A_{28}, A_{29}, A_{33}, A_{34}, A_{37}, A_{39}, A_{41}, A_{44}, A_{45}, A_{47}, A_{48}, A_{50}, A_{51}, A_{54}, A_{55}, A_{57}, A_{59}\}; C_{22k} = 1800 \quad \forall k \in S_{22}$

$S_{32} = \{A_1, A_2, A_5, A_6, A_9, A_{12}, A_{15}, A_{17}, A_{18}, A_{20}, A_{22}, A_{24}, A_{25}, A_{26}, A_{28}, A_{30}, A_{34}, A_{35}, A_{38}, A_{40}, A_{42}, A_{46}, A_{48}, A_{51}, A_{55}, A_{58}\}; C_{32k} = 1700 \quad \forall k \in S_{32}$

$S_{42} = \{A_3, A_4, A_6, A_7, A_{10}, A_{12}, A_{14}, A_{16}, A_{17}, A_{19}, A_{22}, A_{25}, A_{27}, A_{29}, A_{30}, A_{32}, A_{35}, A_{36}, A_{37}, A_{40}, A_{42}, A_{45}, A_{46}, A_{47}, A_{48}, A_{50}, A_{52}, A_{54}, A_{56}, A_{58}, A_{60}\}; C_{42k} = 1600 \quad \forall k \in S_{42}$

$S_{13} = \{A_1, A_2, A_4, A_5, A_8, A_9, A_{11}, A_{13}, A_{15}, A_{17}, A_{18}, A_{20}, A_{21}, A_{23}, A_{24}, A_{28}, A_{31}, A_{33}, A_{35}, A_{37}, A_{39}, A_{41}, A_{43}, A_{45}, A_{48}, A_{51}, A_{52}, A_{53}, A_{54}, A_{55}, A_{57}, A_{59}\}; C_{13k} = 1500 \quad \forall k \in S_{13}$

$S_{23} = \{A_2, A_3, A_5, A_6, A_7, A_{10}, A_{12}, A_{14}, A_{16}, A_{18}, A_{19}, A_{21}, A_{23}, A_{26}, A_{28}, A_{29}, A_{31}, A_{34}, A_{36}, A_{38}, A_{40}, A_{42}, A_{44}, A_{46}, A_{49}, A_{53}, A_{54}, A_{58}, A_{60}\}; C_{23k} = 1400 \quad \forall k \in S_{23}$

$S_{14} = \{A_3, A_4, A_8, A_{10}, A_{11}, A_{14}, A_{17}, A_{22}, A_{24}, A_{25}, A_{26}, A_{29}, A_{32}, A_{34}, A_{38}, A_{41}, A_{44}, A_{46}, A_{49}, A_{51}, A_{53}, A_{56}, A_{59}\}; C_{14k} = 1300 \quad \forall k \in S_{14}$

$S_{24} = \{A_2, A_4, A_7, A_9, A_{12}, A_{14}, A_{16}, A_{18}, A_{21}, A_{23}, A_{26}, A_{28}, A_{29}, A_{31}, A_{33}, A_{35}, A_{37}, A_{39}, A_{42}, A_{45}, A_{47}, A_{49}, A_{52}, A_{54}, A_{56}, A_{59}, A_{60}\}; C_{24k} = 1200 \quad \forall k \in S_{24}$

$S_{34} = \{A_2, A_3, A_5, A_7, A_{10}, A_{13}, A_{15}, A_{17}, A_{19}, A_{22}, A_{25}, A_{26}, A_{29}, A_{32}, A_{33}, A_{36}, A_{38}, A_{40}, A_{43}, A_{44}, A_{46}, A_{48}, A_{51}, A_{53}, A_{55}, A_{58}, A_{60}\}; C_{34k} = 1100 \quad \forall k \in S_{34}$

$S_{15} = \{A_1, A_5, A_6, A_8, A_9, A_{11}, A_{12}, A_{16}, A_{18}, A_{20}, A_{22}, A_{23}, A_{25}, A_{27}, A_{30}, A_{34}, A_{35}, A_{39}, A_{41}, A_{43}, A_{45}, A_{46}, A_{47}, A_{48}, A_{51}, A_{54}, A_{55}, A_{57}, A_{59}\}; C_{15k} = 1000 \quad \forall k \in S_{15}$

$S_{25} = \{A_2, A_4, A_6, A_8, A_{10}, A_{13}, A_{15}, A_{17}, A_{19}, A_{21}, A_{24}, A_{26}, A_{28}, A_{31}, A_{33}, A_{35}, A_{36}, A_{38},$

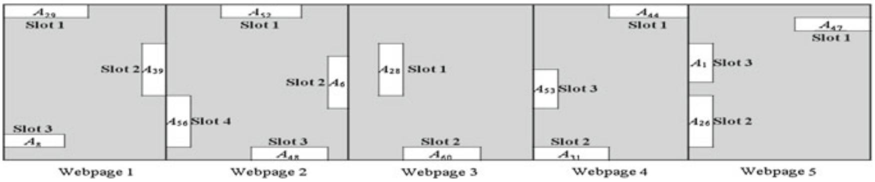

Fig. 2 Display of ads on the news website at time unit $t = 66$

$$A_{40}, A_{42}, A_{44}, A_{46}, A_{47}, A_{49}, A_{50}, A_{53}, A_{56}, A_{58}, A_{59}\}; C_{25\text{k}} = 850 \quad \forall k \in S_{25}$$
$$S_{35} = \{A_1, A_3, A_5, A_7, A_9, A_{11}, A_{12}, A_{14}, A_{18}, A_{20}, A_{21}, A_{23}, A_{25}, A_{27}, A_{30}, A_{32}, A_{34}, A_{37},$$
$$A_{39}, A_{41}, A_{43}, A_{45}, A_{48}, A_{50}, A_{52}, A_{54}, A_{57}, A_{59}\}; C_{35\text{k}} = 800 \quad \forall k \in S_{35}$$

Although different companies may pay different costs for placing their ads on a particular rectangular slot of a specific webpage but for the sake of simplicity, cost for all the ads competing to be placed on any rectangular slot is taken to be same here. Further, minimum and maximum frequencies of the ads are tabulated in Table 1.

When problem (1) is solved using the above data, we get an infeasible solution and hence it is imperative to use GPA to obtain a compromised solution. Aspiration desired on revenue is taken to be ₹ 40,00,000. The compromised solution obtained after applying GPA is given in Table 2.

The revenue generated by this placement of ads comes out to be ₹ 38,47,200. It can be seen from Table 2 that when users access webpage 3 of news website at 18th hour of day 3 i.e., at $t = 66$, ad A_{60} appears in second slot of that webpage. To give a clear picture of how the ads are actually displayed to the users in accordance with the schedule obtained in Table 2, a pictorial representation of the selected ads on the news website at time unit $t = 66$ is shown in Fig. 2.

Table 1 Frequency table

Ads (A_k)	Min. freq. (w_k)	Max. freq. (W_k)	Ads (A_k)	Min. freq. (w_k)	Max. freq. (W_k)	Ads (A_k)	Min. freq. (w_k)	Max. freq. (W_k)	Ads (A_k)	Min. freq. (w_k)	Max. freq. (W_k)	Ads (A_k)	Min. freq. (w_k)	Max. freq. (W_k)	Ads (A_k)	Min. freq. (w_k)	Max. freq. (W_k)
A_1	6	50	A_{11}	6	68	A_{21}	5	55	A_{31}	5	50	A_{41}	5	50	A_{51}	7	50
A_2	7	62	A_{12}	5	50	A_{22}	6	60	A_{32}	4	48	A_{42}	7	55	A_{52}	5	45
A_3	6	55	A_{13}	4	52	A_{23}	7	55	A_{33}	6	52	A_{43}	6	52	A_{53}	4	44
A_4	5	60	A_{14}	6	50	A_{24}	6	52	A_{34}	5	50	A_{44}	7	60	A_{54}	8	65
A_5	7	60	A_{15}	4	45	A_{25}	7	60	A_{35}	7	62	A_{45}	6	55	A_{55}	5	50
A_6	6	58	A_{16}	5	52	A_{26}	7	55	A_{36}	6	58	A_{46}	8	62	A_{56}	4	48
A_7	5	55	A_{17}	7	68	A_{27}	5	60	A_{37}	6	55	A_{47}	7	58	A_{57}	4	50
A_8	5	60	A_{18}	8	70	A_{28}	7	60	A_{38}	7	60	A_{48}	5	50	A_{58}	3	40
A_9	6	62	A_{19}	6	60	A_{29}	6	55	A_{39}	6	55	A_{49}	6	52	A_{59}	7	56
A_{10}	5	60	A_{20}	4	50	A_{30}	4	52	A_{40}	6	58	A_{50}	5	48	A_{60}	5	60

Table 2 Ads allocated to rectangular slot *i* of web page *j* at time unit *t*

Time slots	Rectangular slots														
	Slot 11	Slot 21	Slot 31	Slot 12	Slot 22	Slot 32	Slot 42	Slot 13	Slot 23	Slot 14	Slot 24	Slot 34	Slot 15	Slot 25	Slot 35
1	A_{59}	A_{26}	A_{12}	A_{3}	A_{26}	A_{34}	A_{32}	A_{51}	A_{18}	A_{11}	A_{54}	A_{32}	A_{11}	A_{40}	A_{39}
2	A_{33}	A_{39}	A_{5}	A_{52}	A_{59}	A_{51}	A_{30}	A_{48}	A_{14}	A_{44}	A_{52}	A_{26}	A_{9}	A_{38}	A_{37}
3	A_{59}	A_{35}	A_{1}	A_{47}	A_{55}	A_{35}	A_{29}	A_{45}	A_{12}	A_{41}	A_{49}	A_{22}	A_{8}	A_{36}	A_{34}
4	A_{1}	A_{49}	A_{59}	A_{44}	A_{51}	A_{17}	A_{27}	A_{43}	A_{10}	A_{38}	A_{14}	A_{15}	A_{6}	A_{35}	A_{32}
5	A_{59}	A_{16}	A_{55}	A_{41}	A_{48}	A_{12}	A_{25}	A_{41}	A_{7}	A_{53}	A_{45}	A_{13}	A_{5}	A_{33}	A_{30}
6	A_{40}	A_{33}	A_{51}	A_{19}	A_{51}	A_{6}	A_{22}	A_{39}	A_{6}	A_{49}	A_{39}	A_{10}	A_{1}	A_{28}	A_{27}
7	A_{29}	A_{24}	A_{47}	A_{35}	A_{41}	A_{2}	A_{19}	A_{37}	A_{5}	A_{46}	A_{37}	A_{7}	A_{16}	A_{26}	A_{11}
8	A_{33}	A_{16}	A_{43}	A_{30}	A_{37}	A_{22}	A_{17}	A_{35}	A_{3}	A_{49}	A_{35}	A_{5}	A_{57}	A_{24}	A_{23}
9	A_{18}	A_{26}	A_{40}	A_{25}	A_{33}	A_{17}	A_{16}	A_{33}	A_{42}	A_{38}	A_{33}	A_{3}	A_{55}	A_{21}	A_{27}
10	A_{43}	A_{9}	A_{37}	A_{22}	A_{3}	A_{9}	A_{14}	A_{31}	A_{38}	A_{29}	A_{31}	A_{2}	A_{54}	A_{19}	A_{20}
11	A_{25}	A_{2}	A_{34}	A_{19}	A_{47}	A_{26}	A_{12}	A_{28}	A_{34}	A_{24}	A_{29}	A_{60}	A_{51}	A_{17}	A_{18}
12	A_{57}	A_{50}	A_{29}	A_{13}	A_{39}	A_{58}	A_{10}	A_{39}	A_{29}	A_{17}	A_{28}	A_{58}	A_{48}	A_{15}	A_{14}
13	A_{59}	A_{49}	A_{37}	A_{9}	A_{29}	A_{48}	A_{7}	A_{35}	A_{26}	A_{11}	A_{26}	A_{55}	A_{47}	A_{13}	A_{12}
14	A_{40}	A_{47}	A_{31}	A_{2}	A_{19}	A_{40}	A_{6}	A_{39}	A_{40}	A_{8}	A_{56}	A_{53}	A_{46}	A_{10}	A_{11}
15	A_{23}	A_{44}	A_{24}	A_{27}	A_{59}	A_{34}	A_{4}	A_{31}	A_{34}	A_{3}	A_{52}	A_{51}	A_{45}	A_{8}	A_{9}
16	A_{51}	A_{42}	A_{18}	A_{60}	A_{54}	A_{25}	A_{3}	A_{21}	A_{28}	A_{59}	A_{47}	A_{55}	A_{1}	A_{6}	A_{7}
17	A_{10}	A_{39}	A_{26}	A_{54}	A_{48}	A_{20}	A_{60}	A_{15}	A_{19}	A_{53}	A_{23}	A_{48}	A_{59}	A_{4}	A_{5}
18	A_{44}	A_{37}	A_{5}	A_{49}	A_{44}	A_{15}	A_{58}	A_{33}	A_{14}	A_{49}	A_{21}	A_{44}	A_{57}	A_{2}	A_{3}
19	A_{40}	A_{35}	A_{29}	A_{46}	A_{37}	A_{6}	A_{56}	A_{31}	A_{7}	A_{44}	A_{18}	A_{43}	A_{55}	A_{59}	A_{27}
20	A_{54}	A_{60}	A_{55}	A_{43}	A_{29}	A_{2}	A_{54}	A_{28}	A_{2}	A_{41}	A_{16}	A_{40}	A_{54}	A_{58}	A_{23}
21	A_{27}	A_{56}	A_{37}	A_{39}	A_{23}	A_{55}	A_{52}	A_{55}	A_{60}	A_{22}	A_{14}	A_{38}	A_{51}	A_{56}	A_{20}
22	A_{57}	A_{52}	A_{34}	A_{36}	A_{21}	A_{46}	A_{50}	A_{5}	A_{53}	A_{34}	A_{12}	A_{36}	A_{48}	A_{53}	A_{14}
23	A_{18}	A_{49}	A_{29}	A_{32}	A_{21}	A_{38}	A_{48}	A_{51}	A_{44}	A_{32}	A_{9}	A_{33}	A_{47}	A_{50}	A_{11}

(continued)

Table 2 (continued)

Time slots	Rectangular slots														
	Slot 11	Slot 21	Slot 31	Slot 12	Slot 22	Slot 32	Slot 42	Slot 13	Slot 23	Slot 14	Slot 24	Slot 34	Slot 15	Slot 25	Slot 35
24	A_1	A_{22}	A_{40}	A_{27}	A_{18}	A_{30}	A_{47}	A_{45}	A_{40}	A_{29}	A_4	A_{32}	A_{46}	A_{49}	A_7
25	A_{57}	A_{39}	A_{36}	A_{49}	A_{14}	A_{24}	A_{46}	A_{41}	A_{34}	A_{26}	A_2	A_{29}	A_{45}	A_{47}	A_1
26	A_3	A_{33}	A_{51}	A_{21}	A_{14}	A_{18}	A_{45}	A_{37}	A_6	A_{25}	A_{60}	A_{26}	A_{43}	A_{46}	A_{59}
27	A_{43}	A_{28}	A_{50}	A_2	A_{11}	A_{12}	A_{58}	A_{33}	A_{28}	A_{24}	A_{59}	A_{25}	A_{41}	A_{15}	A_{54}
28	A_{57}	A_{24}	A_{40}	A_{54}	A_8	A_{51}	A_{56}	A_{24}	A_{21}	A_{38}	A_{56}	A_{22}	A_{39}	A_{13}	A_{50}
29	A_{36}	A_{16}	A_{31}	A_{49}	A_6	A_{58}	A_{54}	A_{21}	A_2	A_{17}	A_{54}	A_{19}	A_{35}	A_{10}	A_{41}
30	A_{49}	A_{13}	A_{18}	A_{46}	A_4	A_6	A_{52}	A_{18}	A_{60}	A_{14}	A_{52}	A_{17}	A_{34}	A_8	A_{37}
31	A_{46}	A_9	A_8	A_{43}	A_3	A_5	A_{50}	A_{15}	A_{58}	A_{11}	A_{49}	A_{15}	A_{30}	A_6	A_{32}
32	A_{54}	A_4	A_1	A_{39}	A_8	A_2	A_{48}	A_{11}	A_{49}	A_4	A_{47}	A_{10}	A_{27}	A_4	A_{21}
33	A_{46}	A_2	A_{55}	A_{38}	A_4	A_{12}	A_{47}	A_8	A_{44}	A_8	A_{45}	A_7	A_{25}	A_2	A_{21}
34	A_{44}	A_{60}	A_{45}	A_{35}	A_1	A_6	A_{46}	A_4	A_{42}	A_{10}	A_4	A_5	A_{23}	A_{59}	A_{18}
35	A_{43}	A_{57}	A_{42}	A_{32}	A_3	A_{26}	A_{45}	A_{51}	A_{36}	A_3	A_{39}	A_{33}	A_{22}	A_{58}	A_{12}
36	A_{40}	A_{56}	A_{38}	A_{30}	A_{54}	A_{55}	A_{42}	A_{55}	A_{31}	A_{59}	A_{37}	A_2	A_{20}	A_{56}	A_9
37	A_{38}	A_{54}	A_{34}	A_{27}	A_{51}	A_2	A_{40}	A_{52}	A_{28}	A_{56}	A_{35}	A_{60}	A_{18}	A_{53}	A_5
38	A_{44}	A_{52}	A_{17}	A_{25}	A_{59}	A_{40}	A_{37}	A_{43}	A_{23}	A_{53}	A_{33}	A_{58}	A_{16}	A_{50}	A_1
39	A_{36}	A_{50}	A_{26}	A_{24}	A_{55}	A_{35}	A_{36}	A_{37}	A_{18}	A_{51}	A_{31}	A_{55}	A_{12}	A_{49}	A_{59}
40	A_{32}	A_{49}	A_{23}	A_{22}	A_{50}	A_{30}	A_{35}	A_{31}	A_{14}	A_{49}	A_{29}	A_{53}	A_{11}	A_{47}	A_{57}
41	A_{27}	A_{47}	A_{18}	A_{21}	A_{47}	A_{25}	A_{32}	A_{23}	A_{10}	A_{46}	A_{28}	A_{51}	A_9	A_{46}	A_3
42	A_{27}	A_{44}	A_{14}	A_{19}	A_{44}	A_{35}	A_{30}	A_{18}	A_6	A_{44}	A_{26}	A_{58}	A_8	A_{44}	A_{52}
43	A_{25}	A_{42}	A_{10}	A_{17}	A_{41}	A_{28}	A_{29}	A_{13}	A_5	A_{41}	A_{56}	A_{44}	A_6	A_{42}	A_{50}
44	A_{23}	A_{39}	A_5	A_{15}	A_{39}	A_{20}	A_{27}	A_9	A_3	A_{38}	A_{52}	A_{43}	A_5	A_{40}	A_{48}
45	A_{40}	A_{37}	A_2	A_{13}	A_{37}	A_{15}	A_{25}	A_5	A_{54}	A_{34}	A_{47}	A_{40}	A_1	A_{38}	A_{45}
46	A_{36}	A_4	A_{58}	A_{11}	A_{34}	A_6	A_{22}	A_2	A_{49}	A_{32}	A_{42}	A_{38}	A_{59}	A_{36}	A_{43}

(continued)

Table 2 (continued)

Time slots	Rectangular slots														
	Slot 11	Slot 21	Slot 31	Slot 12	Slot 22	Slot 32	Slot 42	Slot 13	Slot 23	Slot 14	Slot 24	Slot 34	Slot 15	Slot 25	Slot 35
47	A_{32}	A_{33}	A_{53}	A_{9}	A_{33}	A_{1}	A_{19}	A_{54}	A_{19}	A_{29}	A_{37}	A_{36}	A_{57}	A_{35}	A_{41}
48	A_{27}	A_{31}	A_{50}	A_{7}	A_{29}	A_{58}	A_{17}	A_{48}	A_{40}	A_{10}	A_{33}	A_{3}	A_{55}	A_{33}	A_{39}
49	A_{20}	A_{28}	A_{45}	A_{5}	A_{28}	A_{51}	A_{16}	A_{41}	A_{36}	A_{25}	A_{29}	A_{32}	A_{54}	A_{31}	A_{37}
50	A_{15}	A_{16}	A_{51}	A_{3}	A_{27}	A_{55}	A_{14}	A_{35}	A_{31}	A_{24}	A_{23}	A_{29}	A_{51}	A_{28}	A_{34}
51	A_{10}	A_{13}	A_{38}	A_{24}	A_{26}	A_{42}	A_{12}	A_{45}	A_{28}	A_{53}	A_{18}	A_{26}	A_{48}	A_{26}	A_{32}
52	A_{51}	A_{9}	A_{36}	A_{21}	A_{33}	A_{35}	A_{10}	A_{39}	A_{2}	A_{49}	A_{14}	A_{25}	A_{16}	A_{24}	A_{30}
53	A_{49}	A_{6}	A_{31}	A_{17}	A_{16}	A_{28}	A_{7}	A_{33}	A_{21}	A_{44}	A_{9}	A_{22}	A_{12}	A_{21}	A_{27}
54	A_{54}	A_{4}	A_{28}	A_{21}	A_{23}	A_{12}	A_{6}	A_{24}	A_{38}	A_{38}	A_{4}	A_{19}	A_{11}	A_{19}	A_{25}
55	A_{43}	A_{2}	A_{50}	A_{11}	A_{19}	A_{22}	A_{4}	A_{20}	A_{31}	A_{32}	A_{2}	A_{17}	A_{9}	A_{17}	A_{50}
56	A_{33}	A_{60}	A_{45}	A_{5}	A_{16}	A_{18}	A_{3}	A_{15}	A_{26}	A_{25}	A_{52}	A_{15}	A_{8}	A_{15}	A_{45}
57	A_{13}	A_{57}	A_{42}	A_{17}	A_{26}	A_{15}	A_{60}	A_{9}	A_{16}	A_{22}	A_{49}	A_{13}	A_{6}	A_{13}	A_{41}
58	A_{13}	A_{56}	A_{38}	A_{56}	A_{29}	A_{9}	A_{58}	A_{1}	A_{10}	A_{14}	A_{47}	A_{10}	A_{5}	A_{10}	A_{37}
59	A_{3}	A_{54}	A_{36}	A_{47}	A_{27}	A_{5}	A_{56}	A_{59}	A_{54}	A_{10}	A_{45}	A_{7}	A_{1}	A_{8}	A_{32}
60	A_{20}	A_{52}	A_{31}	A_{43}	A_{21}	A_{1}	A_{54}	A_{54}	A_{49}	A_{4}	A_{42}	A_{5}	A_{59}	A_{6}	A_{27}
61	A_{6}	A_{50}	A_{28}	A_{38}	A_{18}	A_{58}	A_{6}	A_{52}	A_{46}	A_{29}	A_{21}	A_{3}	A_{57}	A_{2}	A_{21}
62	A_{44}	A_{49}	A_{24}	A_{11}	A_{16}	A_{51}	A_{4}	A_{48}	A_{40}	A_{56}	A_{26}	A_{2}	A_{55}	A_{59}	A_{18}
63	A_{40}	A_{47}	A_{18}	A_{7}	A_{14}	A_{48}	A_{3}	A_{43}	A_{36}	A_{51}	A_{49}	A_{60}	A_{54}	A_{33}	A_{12}
64	A_{25}	A_{44}	A_{17}	A_{2}	A_{11}	A_{46}	A_{60}	A_{39}	A_{31}	A_{49}	A_{45}	A_{58}	A_{51}	A_{31}	A_{9}
65	A_{10}	A_{42}	A_{12}	A_{56}	A_{8}	A_{42}	A_{58}	A_{33}	A_{2}	A_{46}	A_{35}	A_{55}	A_{48}	A_{28}	A_{5}
66	A_{29}	A_{39}	A_{8}	A_{52}	A_{6}	A_{48}	A_{56}	A_{28}	A_{60}	A_{44}	A_{31}	A_{53}	A_{47}	A_{26}	A_{1}
67	A_{46}	A_{37}	A_{1}	A_{47}	A_{4}	A_{38}	A_{54}	A_{31}	A_{53}	A_{41}	A_{28}	A_{51}	A_{46}	A_{24}	A_{41}
68	A_{32}	A_{35}	A_{59}	A_{44}	A_{3}	A_{35}	A_{52}	A_{21}	A_{44}	A_{38}	A_{26}	A_{53}	A_{45}	A_{21}	A_{39}
69	A_{15}	A_{57}	A_{55}	A_{41}	A_{8}	A_{30}	A_{50}	A_{17}	A_{38}	A_{34}	A_{49}	A_{46}	A_{43}	A_{19}	A_{37}

(continued)

Table 2 (continued)

Time slots	Rectangular slots														
	Slot 11	Slot 21	Slot 31	Slot 12	Slot 22	Slot 32	Slot 42	Slot 13	Slot 23	Slot 14	Slot 24	Slot 34	Slot 15	Slot 25	Slot 35
70	A_{1}	A_{54}	A_{37}	A_{38}	A_{4}	A_{40}	A_{48}	A_{11}	A_{29}	A_{32}	A_{45}	A_{44}	A_{41}	A_{17}	A_{34}
71	A_{57}	A_{50}	A_{47}	A_{36}	A_{1}	A_{22}	A_{47}	A_{9}	A_{23}	A_{4}	A_{39}	A_{43}	A_{39}	A_{15}	A_{32}
72	A_{3}	A_{22}	A_{43}	A_{35}	A_{57}	A_{12}	A_{46}	A_{5}	A_{18}	A_{26}	A_{33}	A_{40}	A_{35}	A_{13}	A_{30}
73	A_{43}	A_{19}	A_{40}	A_{32}	A_{54}	A_{34}	A_{45}	A_{1}	A_{12}	A_{59}	A_{29}	A_{38}	A_{34}	A_{10}	A_{27}
74	A_{38}	A_{31}	A_{37}	A_{30}	A_{51}	A_{28}	A_{42}	A_{59}	A_{58}	A_{53}	A_{28}	A_{36}	A_{30}	A_{8}	A_{25}
75	A_{33}	A_{37}	A_{31}	A_{27}	A_{59}	A_{26}	A_{40}	A_{55}	A_{53}	A_{49}	A_{21}	A_{32}	A_{27}	A_{6}	A_{50}
76	A_{27}	A_{24}	A_{28}	A_{25}	A_{4}	A_{55}	A_{37}	A_{53}	A_{46}	A_{44}	A_{14}	A_{29}	A_{25}	A_{4}	A_{45}
77	A_{23}	A_{26}	A_{24}	A_{24}	A_{50}	A_{48}	A_{36}	A_{2}	A_{42}	A_{38}	A_{12}	A_{26}	A_{23}	A_{2}	A_{41}
78	A_{18}	A_{14}	A_{20}	A_{22}	A_{47}	A_{34}	A_{35}	A_{45}	A_{38}	A_{32}	A_{4}	A_{25}	A_{22}	A_{59}	A_{37}
79	A_{13}	A_{11}	A_{17}	A_{21}	A_{44}	A_{38}	A_{32}	A_{43}	A_{34}	A_{25}	A_{2}	A_{22}	A_{20}	A_{58}	A_{32}
80	A_{6}	A_{2}	A_{12}	A_{19}	A_{41}	A_{34}	A_{30}	A_{41}	A_{29}	A_{22}	A_{59}	A_{19}	A_{18}	A_{56}	A_{27}
81	A_{3}	A_{4}	A_{8}	A_{17}	A_{39}	A_{25}	A_{29}	A_{39}	A_{26}	A_{14}	A_{26}	A_{17}	A_{16}	A_{53}	A_{21}
82	A_{29}	A_{6}	A_{2}	A_{15}	A_{37}	A_{24}	A_{27}	A_{37}	A_{60}	A_{10}	A_{52}	A_{13}	A_{12}	A_{50}	A_{18}
83	A_{54}	A_{60}	A_{59}	A_{13}	A_{34}	A_{20}	A_{25}	A_{35}	A_{53}	A_{4}	A_{49}	A_{10}	A_{11}	A_{49}	A_{12}
84	A_{44}	A_{57}	A_{55}	A_{11}	A_{33}	A_{17}	A_{22}	A_{39}	A_{44}	A_{3}	A_{47}	A_{7}	A_{9}	A_{47}	A_{1}
85	A_{38}	A_{56}	A_{51}	A_{9}	A_{29}	A_{12}	A_{19}	A_{37}	A_{23}	A_{56}	A_{45}	A_{5}	A_{8}	A_{46}	A_{3}
86	A_{32}	A_{54}	A_{47}	A_{7}	A_{28}	A_{6}	A_{17}	A_{35}	A_{31}	A_{51}	A_{42}	A_{3}	A_{6}	A_{44}	A_{1}
87	A_{23}	A_{52}	A_{43}	A_{5}	A_{27}	A_{2}	A_{16}	A_{33}	A_{28}	A_{49}	A_{60}	A_{2}	A_{5}	A_{42}	A_{59}
88	A_{15}	A_{39}	A_{40}	A_{3}	A_{26}	A_{6}	A_{12}	A_{31}	A_{21}	A_{46}	A_{56}	A_{60}	A_{30}	A_{40}	A_{57}
89	A_{8}	A_{35}	A_{2}	A_{22}	A_{29}	A_{5}	A_{10}	A_{28}	A_{18}	A_{44}	A_{52}	A_{58}	A_{27}	A_{38}	A_{54}
90	A_{1}	A_{56}	A_{59}	A_{19}	A_{27}	A_{1}	A_{7}	A_{24}	A_{14}	A_{41}	A_{16}	A_{55}	A_{25}	A_{36}	A_{3}
91	A_{59}	A_{52}	A_{55}	A_{15}	A_{8}	A_{51}	A_{6}	A_{23}	A_{10}	A_{38}	A_{39}	A_{53}	A_{23}	A_{35}	A_{50}
92	A_{51}	A_{49}	A_{24}	A_{11}	A_{55}	A_{48}	A_{4}	A_{21}	A_{6}	A_{34}	A_{35}	A_{51}	A_{22}	A_{33}	A_{48}
93	A_{43}	A_{24}	A_{47}	A_{7}	A_{1}	A_{46}	A_{3}	A_{20}	A_{3}	A_{32}	A_{31}	A_{48}	A_{20}	A_{31}	A_{45}

(continued)

Table 2 (continued)

Time slots	Rectangular slots														
	Slot 11	Slot 21	Slot 31	Slot 12	Slot 22	Slot 32	Slot 42	Slot 13	Slot 23	Slot 14	Slot 24	Slot 34	Slot 15	Slot 25	Slot 35
94	A_{36}	A_{39}	A_{43}	A_{2}	A_{57}	A_{42}	A_{60}	A_{51}	A_{42}	A_{51}	A_{28}	A_{46}	A_{18}	A_{26}	A_{43}
95	A_{49}	A_{33}	A_{40}	A_{60}	A_{54}	A_{55}	A_{14}	A_{28}	A_{36}	A_{26}	A_{21}	A_{44}	A_{16}	A_{24}	A_{41}
96	A_{27}	A_{26}	A_{37}	A_{24}	A_{50}	A_{48}	A_{12}	A_{39}	A_{29}	A_{59}	A_{18}	A_{43}	A_{12}	A_{21}	A_{39}
97	A_{25}	A_{24}	A_{34}	A_{21}	A_{47}	A_{40}	A_{10}	A_{31}	A_{21}	A_{25}	A_{16}	A_{40}	A_{11}	A_{19}	A_{37}
98	A_{23}	A_{19}	A_{29}	A_{17}	A_{44}	A_{35}	A_{7}	A_{20}	A_{16}	A_{24}	A_{14}	A_{38}	A_{9}	A_{17}	A_{34}
99	A_{43}	A_{14}	A_{26}	A_{13}	A_{39}	A_{30}	A_{6}	A_{11}	A_{10}	A_{22}	A_{12}	A_{36}	A_{8}	A_{47}	A_{32}
100	A_{20}	A_{11}	A_{51}	A_{9}	A_{34}	A_{18}	A_{4}	A_{57}	A_{6}	A_{17}	A_{9}	A_{33}	A_{6}	A_{46}	A_{30}
101	A_{32}	A_{9}	A_{45}	A_{5}	A_{29}	A_{25}	A_{3}	A_{54}	A_{3}	A_{14}	A_{2}	A_{32}	A_{5}	A_{44}	A_{27}
102	A_{27}	A_{6}	A_{40}	A_{2}	A_{27}	A_{26}	A_{60}	A_{52}	A_{31}	A_{11}	A_{37}	A_{29}	A_{1}	A_{42}	A_{25}
103	A_{20}	A_{4}	A_{36}	A_{56}	A_{23}	A_{22}	A_{58}	A_{45}	A_{28}	A_{10}	A_{33}	A_{26}	A_{59}	A_{40}	A_{50}
104	A_{15}	A_{2}	A_{29}	A_{52}	A_{19}	A_{20}	A_{56}	A_{41}	A_{23}	A_{8}	A_{60}	A_{25}	A_{57}	A_{38}	A_{45}
105	A_{10}	A_{60}	A_{24}	A_{47}	A_{16}	A_{18}	A_{54}	A_{37}	A_{21}	A_{4}	A_{59}	A_{22}	A_{55}	A_{36}	A_{18}
106	A_{6}	A_{57}	A_{17}	A_{44}	A_{11}	A_{17}	A_{52}	A_{33}	A_{58}	A_{3}	A_{56}	A_{19}	A_{54}	A_{35}	A_{11}
107	A_{1}	A_{56}	A_{10}	A_{41}	A_{6}	A_{15}	A_{50}	A_{20}	A_{54}	A_{59}	A_{54}	A_{17}	A_{51}	A_{33}	A_{5}
108	A_{29}	A_{54}	A_{1}	A_{38}	A_{1}	A_{12}	A_{48}	A_{23}	A_{53}	A_{56}	A_{52}	A_{15}	A_{48}	A_{31}	A_{1}
109	A_{57}	A_{52}	A_{59}	A_{36}	A_{59}	A_{9}	A_{47}	A_{20}	A_{49}	A_{53}	A_{49}	A_{13}	A_{47}	A_{28}	A_{57}
110	A_{54}	A_{50}	A_{53}	A_{35}	A_{55}	A_{15}	A_{46}	A_{17}	A_{46}	A_{51}	A_{9}	A_{10}	A_{46}	A_{26}	A_{52}
111	A_{51}	A_{49}	A_{47}	A_{32}	A_{51}	A_{12}	A_{45}	A_{13}	A_{6}	A_{49}	A_{45}	A_{7}	A_{45}	A_{24}	A_{50}
112	A_{49}	A_{47}	A_{40}	A_{30}	A_{59}	A_{9}	A_{42}	A_{9}	A_{26}	A_{46}	A_{42}	A_{5}	A_{43}	A_{21}	A_{48}
113	A_{49}	A_{44}	A_{36}	A_{27}	A_{55}	A_{6}	A_{40}	A_{5}	A_{40}	A_{44}	A_{29}	A_{3}	A_{41}	A_{19}	A_{45}
114	A_{44}	A_{42}	A_{29}	A_{25}	A_{50}	A_{1}	A_{37}	A_{2}	A_{38}	A_{41}	A_{49}	A_{2}	A_{39}	A_{17}	A_{43}
115	A_{40}	A_{39}	A_{23}	A_{24}	A_{47}	A_{35}	A_{36}	A_{57}	A_{36}	A_{38}	A_{45}	A_{58}	A_{35}	A_{15}	A_{41}
116	A_{36}	A_{37}	A_{26}	A_{22}	A_{44}	A_{30}	A_{35}	A_{53}	A_{34}	A_{34}	A_{37}	A_{55}	A_{34}	A_{13}	A_{39}
117	A_{32}	A_{35}	A_{14}	A_{21}	A_{41}	A_{58}	A_{32}	A_{41}	A_{10}	A_{32}	A_{33}	A_{53}	A_{30}	A_{10}	A_{37}

(continued)

Table 2 (continued)

Time slots	Rectangular slots														
	Slot 11	Slot 21	Slot 31	Slot 12	Slot 22	Slot 32	Slot 42	Slot 13	Slot 23	Slot 14	Slot 24	Slot 34	Slot 15	Slot 25	Slot 35
118	A_{25}	A_{54}	A_{10}	A_{19}	A_{39}	A_{55}	A_{29}	A_{39}	A_{40}	A_{3}	A_{29}	A_{51}	A_{27}	A_{8}	A_{34}
119	A_{20}	A_{50}	A_{5}	A_{17}	A_{37}	A_{25}	A_{27}	A_{37}	A_{36}	A_{46}	A_{26}	A_{53}	A_{25}	A_{6}	A_{32}
120	A_{15}	A_{24}	A_{2}	A_{15}	A_{34}	A_{48}	A_{25}	A_{35}	A_{29}	A_{51}	A_{23}	A_{46}	A_{23}	A_{2}	A_{30}
121	A_{8}	A_{42}	A_{58}	A_{11}	A_{33}	A_{46}	A_{22}	A_{33}	A_{5}	A_{8}	A_{21}	A_{40}	A_{22}	A_{59}	A_{27}
122	A_{3}	A_{37}	A_{53}	A_{11}	A_{29}	A_{42}	A_{19}	A_{31}	A_{19}	A_{41}	A_{18}	A_{43}	A_{20}	A_{58}	A_{25}
123	A_{36}	A_{31}	A_{18}	A_{7}	A_{28}	A_{58}	A_{17}	A_{28}	A_{18}	A_{34}	A_{16}	A_{40}	A_{18}	A_{56}	A_{52}
124	A_{27}	A_{26}	A_{14}	A_{5}	A_{27}	A_{24}	A_{16}	A_{24}	A_{16}	A_{29}	A_{14}	A_{38}	A_{46}	A_{53}	A_{48}
125	A_{20}	A_{22}	A_{10}	A_{3}	A_{48}	A_{18}	A_{14}	A_{23}	A_{14}	A_{24}	A_{12}	A_{33}	A_{45}	A_{50}	A_{43}
126	A_{13}	A_{39}	A_{5}	A_{25}	A_{44}	A_{38}	A_{12}	A_{21}	A_{12}	A_{17}	A_{9}	A_{32}	A_{43}	A_{49}	A_{39}
127	A_{6}	A_{16}	A_{2}	A_{22}	A_{37}	A_{34}	A_{10}	A_{20}	A_{10}	A_{11}	A_{7}	A_{29}	A_{41}	A_{47}	A_{34}
128	A_{1}	A_{31}	A_{58}	A_{19}	A_{29}	A_{28}	A_{7}	A_{18}	A_{7}	A_{8}	A_{4}	A_{26}	A_{39}	A_{46}	A_{30}
129	A_{54}	A_{24}	A_{53}	A_{15}	A_{23}	A_{26}	A_{22}	A_{17}	A_{6}	A_{3}	A_{2}	A_{25}	A_{35}	A_{44}	A_{23}
130	A_{46}	A_{19}	A_{50}	A_{39}	A_{18}	A_{51}	A_{19}	A_{15}	A_{5}	A_{59}	A_{60}	A_{22}	A_{34}	A_{42}	A_{20}
131	A_{40}	A_{14}	A_{45}	A_{36}	A_{11}	A_{46}	A_{17}	A_{13}	A_{3}	A_{53}	A_{59}	A_{19}	A_{30}	A_{40}	A_{14}
132	A_{33}	A_{11}	A_{42}	A_{32}	A_{4}	A_{30}	A_{16}	A_{11}	A_{58}	A_{51}	A_{56}	A_{17}	A_{27}	A_{38}	A_{11}
133	A_{25}	A_{6}	A_{38}	A_{27}	A_{1}	A_{38}	A_{14}	A_{9}	A_{53}	A_{49}	A_{54}	A_{15}	A_{25}	A_{36}	A_{7}
134	A_{20}	A_{2}	A_{36}	A_{22}	A_{57}	A_{34}	A_{12}	A_{8}	A_{46}	A_{46}	A_{54}	A_{13}	A_{23}	A_{35}	A_{3}
135	A_{13}	A_{60}	A_{31}	A_{19}	A_{54}	A_{25}	A_{10}	A_{5}	A_{42}	A_{44}	A_{52}	A_{10}	A_{22}	A_{2}	A_{5}
136	A_{1}	A_{56}	A_{28}	A_{15}	A_{48}	A_{24}	A_{7}	A_{4}	A_{38}	A_{41}	A_{49}	A_{7}	A_{20}	A_{2}	A_{1}
137	A_{3}	A_{52}	A_{23}	A_{11}	A_{44}	A_{20}	A_{6}	A_{2}	A_{14}	A_{38}	A_{47}	A_{5}	A_{18}	A_{2}	A_{57}
138	A_{54}	A_{49}	A_{20}	A_{7}	A_{41}	A_{17}	A_{4}	A_{17}	A_{12}	A_{34}	A_{45}	A_{3}	A_{16}	A_{2}	A_{52}
139	A_{43}	A_{44}	A_{17}	A_{2}	A_{37}	A_{12}	A_{3}	A_{13}	A_{10}	A_{32}	A_{42}	A_{2}	A_{12}	A_{2}	A_{50}
140	A_{49}	A_{39}	A_{12}	A_{60}	A_{33}	A_{6}	A_{48}	A_{9}	A_{6}	A_{29}	A_{39}	A_{60}	A_{11}	A_{2}	A_{48}
141	A_{46}	A_{35}	A_{8}	A_{54}	A_{28}	A_{2}	A_{58}	A_{53}	A_{5}	A_{26}	A_{37}	A_{58}	A_{9}	A_{2}	A_{45}
142	A_{44}	A_{47}	A_{1}	A_{49}	A_{26}	A_{6}	A_{56}	A_{1}	A_{3}	A_{59}	A_{35}	A_{55}	A_{8}	A_{2}	A_{43}
143	A_{43}	A_{39}	A_{59}	A_{46}	A_{21}	A_{1}	A_{54}	A_{59}	A_{6}	A_{3}	A_{33}	A_{53}	A_{6}	A_{44}	A_{1}

(continued)

Table 2 (continued)

Time slots	Rectangular slots														
	Slot 11	Slot 21	Slot 31	Slot 12	Slot 22	Slot 32	Slot 42	Slot 13	Slot 23	Slot 14	Slot 24	Slot 34	Slot 15	Slot 25	Slot 35
144	A_{40}	A_{31}	A_{55}	A_{43}	A_{18}	A_{55}	A_{52}	A_{55}	A_{2}	A_{3}	A_{31}	A_{51}	A_{5}	A_{42}	A_{1}
145	A_{38}	A_{26}	A_{36}	A_{39}	A_{14}	A_{51}	A_{50}	A_{53}	A_{3}	A_{26}	A_{29}	A_{48}	A_{1}	A_{40}	A_{9}
146	A_{36}	A_{22}	A_{47}	A_{38}	A_{8}	A_{48}	A_{60}	A_{51}	A_{54}	A_{3}	A_{28}	A_{46}	A_{59}	A_{38}	A_{3}
147	A_{33}	A_{16}	A_{43}	A_{36}	A_{4}	A_{20}	A_{47}	A_{2}	A_{53}	A_{3}	A_{26}	A_{44}	A_{57}	A_{36}	A_{1}
148	A_{32}	A_{13}	A_{38}	A_{35}	A_{3}	A_{42}	A_{46}	A_{43}	A_{49}	A_{3}	A_{23}	A_{43}	A_{55}	A_{35}	A_{1}
149	A_{29}	A_{11}	A_{26}	A_{32}	A_{57}	A_{58}	A_{45}	A_{41}	A_{46}	A_{8}	A_{21}	A_{40}	A_{54}	A_{33}	A_{1}
150	A_{57}	A_{9}	A_{45}	A_{30}	A_{50}	A_{1}	A_{42}	A_{48}	A_{44}	A_{3}	A_{18}	A_{38}	A_{51}	A_{31}	A_{1}
151	A_{51}	A_{6}	A_{40}	A_{25}	A_{44}	A_{58}	A_{37}	A_{45}	A_{26}	A_{59}	A_{16}	A_{36}	A_{48}	A_{26}	A_{1}
152	A_{46}	A_{4}	A_{34}	A_{24}	A_{39}	A_{20}	A_{36}	A_{43}	A_{49}	A_{53}	A_{14}	A_{33}	A_{47}	A_{24}	A_{25}
153	A_{1}	A_{2}	A_{28}	A_{22}	A_{33}	A_{34}	A_{35}	A_{41}	A_{44}	A_{49}	A_{12}	A_{32}	A_{46}	A_{21}	A_{9}
154	A_{32}	A_{60}	A_{23}	A_{21}	A_{28}	A_{24}	A_{32}	A_{39}	A_{40}	A_{44}	A_{9}	A_{29}	A_{45}	A_{19}	A_{18}
155	A_{43}	A_{57}	A_{17}	A_{19}	A_{34}	A_{55}	A_{30}	A_{37}	A_{36}	A_{38}	A_{4}	A_{26}	A_{43}	A_{17}	A_{12}
156	A_{36}	A_{56}	A_{10}	A_{17}	A_{27}	A_{42}	A_{29}	A_{35}	A_{31}	A_{32}	A_{2}	A_{25}	A_{41}	A_{15}	A_{9}
157	A_{29}	A_{54}	A_{12}	A_{15}	A_{21}	A_{46}	A_{27}	A_{33}	A_{29}	A_{25}	A_{60}	A_{22}	A_{39}	A_{13}	A_{5}
158	A_{20}	A_{52}	A_{10}	A_{13}	A_{14}	A_{34}	A_{25}	A_{31}	A_{28}	A_{22}	A_{59}	A_{19}	A_{35}	A_{10}	A_{1}
159	A_{54}	A_{50}	A_{5}	A_{11}	A_{44}	A_{28}	A_{22}	A_{28}	A_{26}	A_{14}	A_{56}	A_{15}	A_{34}	A_{8}	A_{57}
160	A_{51}	A_{49}	A_{1}	A_{9}	A_{41}	A_{30}	A_{19}	A_{24}	A_{23}	A_{10}	A_{54}	A_{60}	A_{59}	A_{6}	A_{52}
161	A_{59}	A_{47}	A_{58}	A_{7}	A_{3}	A_{24}	A_{17}	A_{23}	A_{21}	A_{4}	A_{52}	A_{55}	A_{57}	A_{4}	A_{48}
162	A_{57}	A_{44}	A_{53}	A_{5}	A_{33}	A_{18}	A_{16}	A_{21}	A_{19}	A_{3}	A_{49}	A_{53}	A_{55}	A_{2}	A_{43}
163	A_{10}	A_{42}	A_{50}	A_{3}	A_{29}	A_{15}	A_{32}	A_{20}	A_{18}	A_{56}	A_{47}	A_{51}	A_{54}	A_{59}	A_{39}
164	A_{36}	A_{35}	A_{45}	A_{49}	A_{28}	A_{9}	A_{30}	A_{18}	A_{16}	A_{51}	A_{45}	A_{48}	A_{51}	A_{58}	A_{34}
165	A_{13}	A_{33}	A_{42}	A_{46}	A_{27}	A_{5}	A_{29}	A_{17}	A_{14}	A_{26}	A_{42}	A_{46}	A_{48}	A_{56}	A_{30}
166	A_{23}	A_{31}	A_{38}	A_{43}	A_{26}	A_{1}	A_{27}	A_{15}	A_{12}	A_{41}	A_{39}	A_{44}	A_{47}	A_{53}	A_{25}
167	A_{15}	A_{28}	A_{34}	A_{38}	A_{48}	A_{55}	A_{25}	A_{13}	A_{10}	A_{34}	A_{37}	A_{43}	A_{46}	A_{50}	A_{32}
168	A_{13}	A_{26}	A_{29}	A_{35}	A_{44}	A_{2}	A_{22}	A_{11}	A_{2}	A_{29}	A_{35}	A_{40}	A_{45}	A_{49}	A_{23}

(continued)

4 Conclusion

In this paper, we have formulated a web ad scheduling problem to determine an optimal placement of ads that compete to be placed on rectangular slots in a given planning horizon on the various webpages of a news website in order to maximize the revenue generated from ads. The optimization model is a 0-1 linear programming model and restricts selection of an ad on the same webpage more than once at any instant of time. Problem is programmed on LINGO software to get the optimal solution. In case the problem results in an infeasible solution, goal programming method is used to solve the problem. A case study is presented in the paper to show the application of the problem.

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